The three flavour chiral phase transition with an improved quark and gluon action in lattice QCD [*]

A. Peikert with F. Karsch, E. Laermann and B. Sturm

Fakultät für Physik, Universität Bielefeld, 33501 Bielefeld, Germany

The finite-temperature chiral phase transition is investigated for three flavours of staggered quarks on a lattice of temporal extent $N_\tau = 4$. In the simulation we use an improved fermion action which reduces rotational symmetry breaking of the quark propagator (p4-action), include fat-links to improve the flavour symmetry and use the tree level improved (1,2) gluon action. We study the nature of the phase transition for quark masses of $ma = 0.025$, $ma = 0.05$ and $ma = 0.1$ on lattices with spatial sizes of 8^3 and 16^3.

1. INTRODUCTION

QCD predicts a finite-temperature chiral phase transition from a hadronic to a plasma phase. The nature of this phase transition depends on the number of flavours and the quark masses. Results from linear sigma models suggest[1] a second order transition for two and a first order transition for three degenerate quarks in the limit of zero quark mass. The second order line
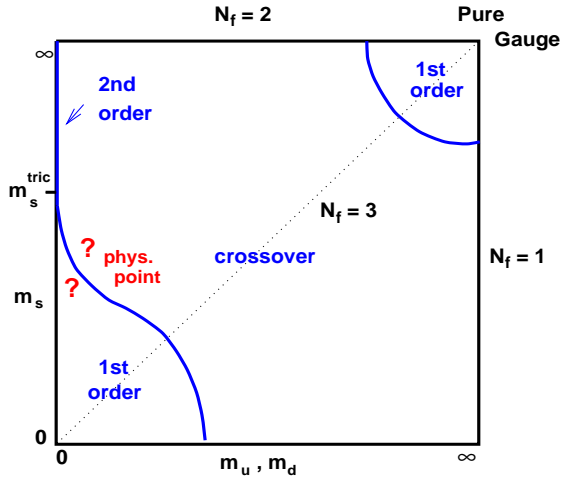

Figure 1. The nature of the finite-temperature QCD phase transition as a function of $m_{u,d}a$ and $m_s a$.
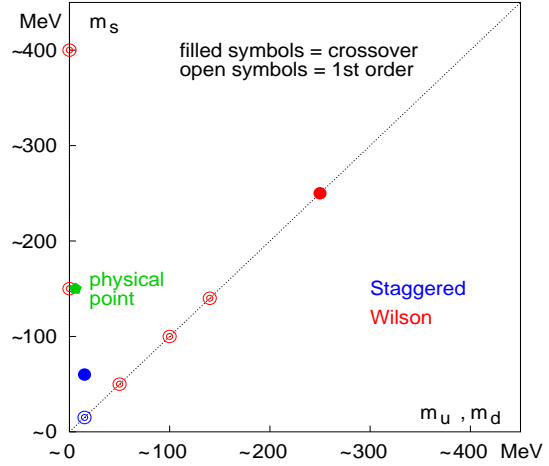

Figure 2. Simulation results for the order of the QCD phase transition for $N_f = 2+1$ staggered[2] and Wilson[3] fermions for $N_\tau = 4$.

in figure 1 runs into a tricritical point and then continues according to $m_u, m_d \propto \left(m_s^{tric} - m_s\right)^{\frac{5}{2}}$ separating the first order from the crossover region. In figure 2 these predictions are compared with results from lattice simulations using staggered[2] and Wilson[3] fermions. These data lead to different results for the order of the phase transition at the physical point of two light up and down quarks and one heavier strange quark. Both simulations may however suffer from large deviations from the continuum limit which can be reduced by using an improved action.

2. THE ACTION

We used the 1×2 action for the gauge fields

$$S_G = \beta \sum_{x,\nu>\mu} \frac{5}{3}\left(1 - \frac{1}{N}\mathrm{ReTr}\ \boxed{}_{\mu\nu}(x)\right) -$$

$$\frac{1}{6}\left(1 - \frac{1}{2N}\mathrm{ReTr}\left(\boxed{}_{\mu\nu}(x) + \boxed{}_{\mu\nu}(x)\right)\right),$$

which leads to an improved finite cut-off behaviour for quantities like latent heat, surface tension and pressure in pure SU(3) gauge simulations[4]. For the fermion fields we used the p4-action which is constructed from 1-link and L-shaped 3-link paths in the derivative under the constraint that the free quark propagator should be rotationally invariant up to order p^4. The appropriate coefficients have been calculated at tree-level and to one-loop-order[5]. The off-diagonal part of the fermion matrix is then given by

$$M[U]_{ij} = \eta_i\Bigg\{\left(\frac{3}{8} + g^2\ \frac{N^2-1}{2N}\ 0.0165\right)\ A[U]_{ij}$$

$$+ \left(\frac{1}{48} - g^2\ \frac{N^2-1}{2N}\ \frac{1}{6}\ 0.0165\right)\ \frac{1}{2}\ B[U]_{ij}\Bigg\}$$

$$A[U]_{ij} = {}_{j}\xleftarrow{\ i\ }{}_{j}$$

$$B[U]_{ij} = \ \raisebox{0pt}{$\;$} + \raisebox{0pt}{$\;$} + \raisebox{0pt}{$\;$} + \raisebox{0pt}{$\;$}\ .$$

The improvement of rotational symmetry leads to a fast approach to the continuum ideal gas limit as can be seen for the free energy in figure 3. Both p4 and Naik[1]-action are much closer to the ideal gas value already at small N_τ than the standard staggered action. In figure 4 the effect of improving the p4-action to one-loop order is shown.

In addition fat-links[6] are used in the one-link derivative $A[U]_{ij}$ which leads to an improved

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[1]The Naik action also has a propagator invariant up to order p^4.

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flavour symmetry indicated by a reduced pion splitting. This effect has been seen for the tree-level p4-action in a quenched simulation[7].
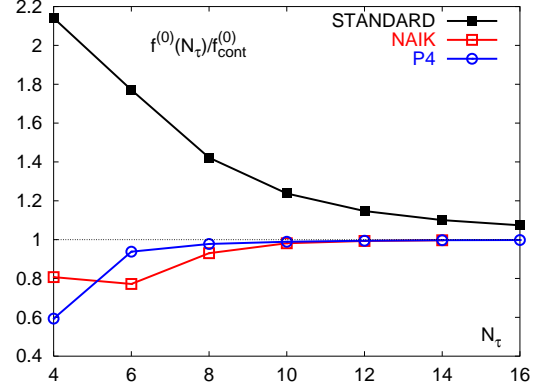

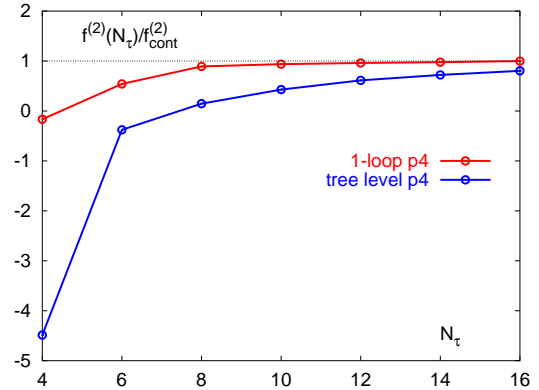Figure 3. The fermion free energy in the ideal gas limit.

Figure 4. Corrections to the fermion free energy in one-loop order.

3. THE SIMULATION

We have performed simulations with the tree-level p4 action with a fat-weight $\omega = 0.2$ using the Hybrid R algorithm with a step size $\Delta\tau < m_q a/2$ and a trajectory length of 0.8. We simulated three degenerate quark flavours of mass $m_q a = 0.1, 0.05$ and 0.025 on lattice sizes of $8^3 \times 4$ and $16^3 \times 4$.

3.1. ROTATIONAL SYMMETRY

To test the effect of improving the rotational symmetry of the action we extracted the potential

from Polyakov-Loop correlations according to

$$V(R) = -\frac{1}{N_\tau} \ln \langle PLC(R) \rangle$$

$$PLC(R) = \langle L^\dagger(x)L(y) \rangle, \quad \text{with } R = |x - y|.$$

The potential in figure 5 clearly shows string breaking for both actions. At short distances the deviations from rotational symmetry are smaller for the improved than for the standard action.
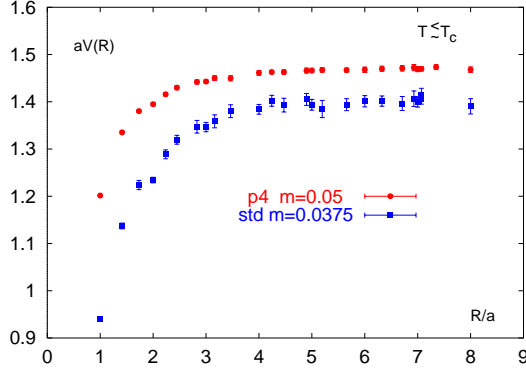

Figure 5. The potential from Polyakov loop correlations for the p4- and standard action[8].

3.2. CHIRAL PHASE TRANSITION

In figure 6 we present our measurement of the chiral condensate. There is no sign of a discontinuous behaviour in $\langle \bar{\Psi}\Psi \rangle$. Likewise there is no two state signal in the time-histories. The finite-size effects are small since the
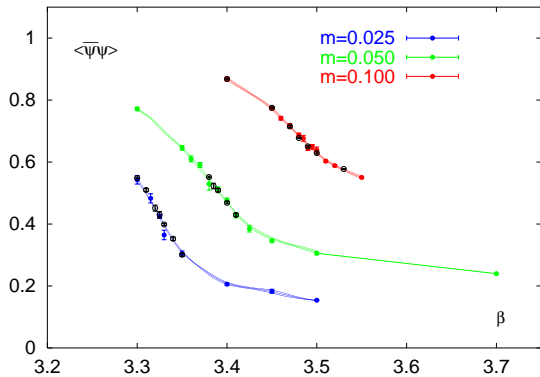

Figure 6. The chiral condensate on $16^3 \times 4$ (black dots) and on $8^3 \times 4$ (other symbols and curves).
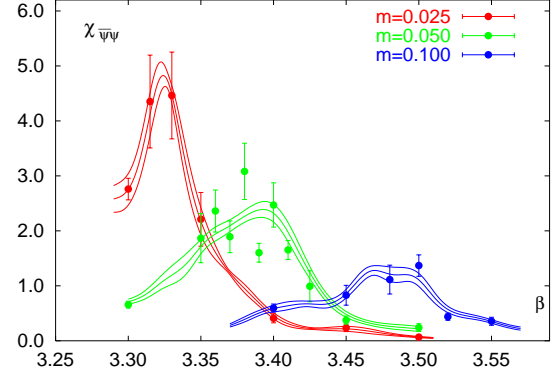

Figure 7. The susceptibility of the chiral condensate on $8^3 \times 4$.

results from the $16^3 \times 4$ lattice lie perfectly on the $8^3 \times 4$ curve. The increase of the peak height of the susceptibility with smaller quark mass in figure 7 qualitatively agrees with what has been found for the two flavour case.

We conclude that there is no first order transition for bare quark masses of $m_q a = 0.1$, 0.05 and 0.025. This confirms the earlier findings with the standard staggered fermion action that a regime of first order transitions only starts for quite light quarks. On a qualitative level our calculation with an improved action gives the impression that the first order regime is pushed to even smaller quark mass values. This, however, requires a more detailed analysis of hadron masses in order to set a physical scale.

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